A Synopsis
of the proposed thesis entitled

'SOME NEW RESULTS ON STRUCTURE OF ZERO DIVISOR GRAPHS'

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Abstract:

In Mathematics, Graph theory is the study of graphs, which are mathematically structures, used to model pair wise relations between objects. A graph in this context is made up of vertices, nodes or points which are connected by edges, arcs or lines.

In this research work, we study of structure of zero divisor graphs, graphs of rings, graphs of commutative and non-commutative rings, graphs of homomorphism of rings and we prove some new results.

1. Introduction :

Let R be a commutative ring with unity and let $Z(R)$ be its set of zero-divisors. We associate a graph $\Gamma(R)$ to R with vertices

$$Z(R)^* = Z(R)-\{0\}$$

the set of nonzero zero-divisors of R, and for distinct $x, y \in Z(R)^*$, the vertices x and y are adjacent iff

$$x \neq y \quad and \quad x \,.\, y = 0.$$

Thus $\Gamma(R)$ is the empty graph if and only if R is an integral domain.

The zero divisor graph gives the interplay of ring-theoretic properties of R with graph-theoretic properties of $\Gamma(R)$.

1.1: Definitions:

a. Commutative Ring:

A ring **R** is called commutative ring if for all $s,r \in \boldsymbol{R}$

$$s.r = r.s$$

b. Ring with unity:

A ring **R** is called ring with unity if for all $s \in \boldsymbol{R}$ **,** there exist $e \in \boldsymbol{R}$ such that $e.s = s.e = s$

c. Zero Divisor:

An element $a \in \boldsymbol{R}$ is called zero divisor of **R** if there exist $0 \neq b \in \boldsymbol{R}$ such that $a.b = 0$.

d. Noetherian Rings:

A Notherian ring is a ring that satisfies the ascending chain condition on ideals; that is, given any chain of ideal;

$$I_1 \subseteq I_2 \subseteq .... \subseteq I_{k-1} \subseteq I_k \subseteq I_{k+1} \subseteq ....$$

There exist an n such that;

$$I_n = I_{n+1} = ...$$

e. Simple Graph:

A Simple Graph is one with no loops on a vertex and no multiple edges between a pair of vertices.

f. ***Complete Graph***:

A graph is said to be complete if every vertex in the graph is adjacent to every other vertex in the graph .The notation for a complete graph on n vertices is Kn.

g. Bipartite Graph:

A graph is bipartite (respectively, r-*partite*) if the vertices can be partitioned into two (resp., r) disjoint subsets so that every edge has one vertex in each subset (resp., every edge joins vertices in distinct subsets).

h. Complete Bipartite Graph:

A complete bipartite is a bipartite graph such that every vertex in one partitioning subset is adjacent to every vertex in the other partitioning subset. If the subsets have cardinality m, and n, then this graph is denoted by $K_{m.n}$.

i. ***Coloring of Graph:***

A coloring of a graph is an assignment of colors to the vertices of a graph in a way such that no two adjacent vertices receive the same color. The minimal number of colors needed to color a graph $\Gamma(R)$ is denoted by $\varkappa(\Gamma(R))$ and called chromatic number.

j. Connected Graph:

A graph is said to be connected if there is a path between every pair of vertices of the graph.

k. Directed Graph:

A directed graph is a pair *(V,E)* of disjoint sets of vertices and edges together with two maps

$$Init : W \rightarrow E \text{ and } ter : E \rightarrow V$$

assigning to every *edge (e)* an initial vertex i*nit* (e) and a terminal vertex *ter (e).* The edge *e* is said to be directed from *init(e)* to *ter(e).*

2. *Review Of Literature:*

Here we investigate mainly the coloring problem of the zero divisor graph of various commutative algebraic structures such as commutative groupoid, commutative semi group, distributive lattice, commutative semiring and commutative loopring.

There are three types of zero divisor graph.

2.1 Beck's Zero Divisor Graph

2.2 Anderson and Livingston's Zero Divisor Graph

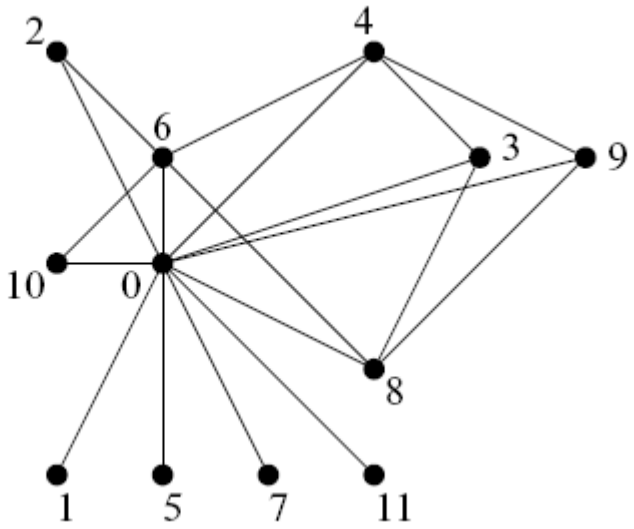
2.3 Mulay's Zero Divisor Graph

2.1 Beck's Zero Divisor Graph:

The idea of a zero divisor *graph was* first given by Beck in 1988 [1] where he was mainly interested in coloring. He took all the elements of ring as vertices of the graph .Thus the structure were not so interesting.

e.g. the graph of Z/12Z is given by

$G(\mathbb{Z}/12\mathbb{Z})$

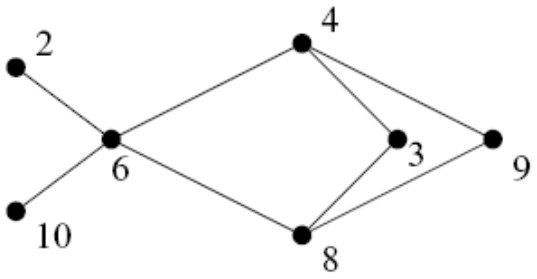


2.2Anderson and Livingston's Zero Divisor Graph:

After Beck ,Anderson and Livingston in1999 [2] gives more simple form of zero divisor graph which includes simple graph as that of Beck but the edges are the only zero divisor of the ring.

e.g. the graph of Z/12Z is given by

$\Gamma(\mathbb{Z}/12\mathbb{Z})$

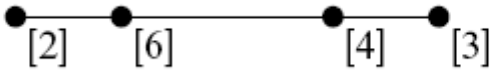


2.3Mulay's Zero Divisor Graph:

After Anderson and Livingston S. B. Mulay [3] gives the next zero divisor graph associated to a ring in 2002. He partitioned the zero divisor into equivalence classes and thus the graph has vertex set equal to the set of equivalence class.

e.g. the graph of Z/12Z is given by

$\Gamma_E(\mathbb{Z}/12\mathbb{Z})$



Shane Patrick Redmond:

In 2001 Shane Patrick Redmond [4] extends the definition of the zero-divisor graph to a noncommutative ring in several ways. After that he

introduced for a given commutative ring **R** and ideal **I** of **R**, the notion of an ideal-based graph. This was also an undirected graph with vertex set,

*{x ϵ **R-I** I x.y ϵ **I** for some y ϵ **R-I**},*

where distinct vertices x and y are adjacent if and only if

*x.y ϵ **I**.*

Adrianna Guillory:

In 2004 Adrianna Guillory [5] in his works realize that it is simple to construct the zero divisor graphs for a given commutative ring with unity, but the opposite endeavor is more complicated, i.e. to supply a commutative ring for the given graphs .

Darrin Weber :

In his work[7] he consider a finite commutative ring R, the well-studied zero-divisor graph $\Gamma(R)$ and the compressed zero-divisor graph $\Gamma_c(R)$ of R and a newly-defined graphical structure — the zero-divisor lattice $\Lambda(R)$ of R.

Results are also extended to non commutative rings.

3. Aims and Objectives of Research:

- To study different commutative and non commutative rings and their relevant undirected and directed zero divisor graphs respectively.
- To make the relation between the properties of ring and properties of graphs.
- To study the basic properties such as connectivity, diameter and girth of the zero divisor graphs.
- To established ideal- based zero divisor graphs.
- To calculate diameter and girths of zero divisor graphs of commutative ring and compare the result with the graph of non commutative ring.
- To evaluate degree of vertex of zero divisor graph and relate its properties with ring and the ring theoretic properties.
- To find different homomorphism of ring **R** and find their respective zero divisor graph and determine the relation between graph of **R** and corresponding subgraphs of relevant zero divisor graphs.

4. Methodology:

For the above mentioned study, first of all we have to study properties of ring theory and graph theory in detail. For this purpose we will make use of various basic books on this topics. We will collect all the related material from online sources. After that we take the help of research paper which were published earlier in various national and international journals to understand the parameters.

We will take small structured commutative rings and find their zero divisor graph. After this we will move towards large structure of commutative ring and if the complications are arrived we will make use of different mathematical software's such as 'Mathematica'.

We extend our study for small and large structured non commutative rings and find their zero divisor graphs. After this we will try to compare the graphs of commutative and non commutative ring.

5. Importance of study:

Graph theory is used to study molecules in Chemistry and Physics, three-dimensional structure of complicated simulated atomic structure can be studied quantitatively by gathering statistics on graph-theoretic properties related to the topology of the atoms. This approach is especially used in Computer processing of molecular structures, ranging from chemical editors to database searching. Similarly in computational neuroscience graphs can be used to represent functional connections.

The development of algorithms to handle graphs is major interest in computer science. By knowing the applications and use of graph theory in every field and science, we decided to study 'Graph theory' to develop new results.

Zero divisor graph is nothing but the relation between ring theory and graph theory. So by using this we can able to understand most complicated structure of ring (which are not easy to visualize) with the help of graph. In this way the properties of ring can be easily manipulated with the properties of graph.

After this we will able to relate the properties of graph such as girth, chromatic no., connectivity, diameter, sub-graph, etc with the ring theoretic properties such as ideal, sub-ring, etc.

6. Plan of Research:

First Year:

- To make study all the basics from ring theory and graph theory which will be required.
- To collect the related materials from online sources.
- To refer related papers which were published earlier on this topic.
- To study various examples of rings and find their zero divisors.
- To study graph theory in details.
- To try to make the relation between properties of ring and properties of graph.

Second Year:

- To study the zero divisor graph of finite commutative ring in details, Z_n i.e. **Z**n , **Z**n[i] , polynomial rings ,etc.
- To extend our study for large values n by using mathematical software's.
- We will move towards different non commutative rings and their zero divisor graphs.
- To publish papers in Conference and journals.
- To try to form different non commutative ideal-based zero divisor graph.
- To complete the course work and course work examination.

Third year:

- To the application of zero divisor graphs.
- To publish papers in national/ international journal.

Forth year:

- To submit final synopsis and thesis to the university.

7. References:

[1] : Beck, "Coloring of a commutative ring", J. Algebra, 116 (1988), 208-226.

[2] : D. F. Anderson and P. F. Livingston, "The zero-divisor graph of a commutative ring" , J. Algebra, 217 (1999), 437-447.

[3] : S. B. Mulay, Cycles and symmetries of zero-divisors, Comm. Algebra, 30(7) (2002), 3533-3558.

[4] : Redmond, Shane Patrick, "Generalizations of the Zero- Divisor Graph of a Ring" Ph.D. diss., University of Tennessee,2001. http://trace.tennessee.edu/utk_graddiss/1504

[5] : Adrianna Guillory, Mhellazo, Laura Mondello and Thomas Naugle, "Realizing Zero Divisor Graph" , University of Mississippi 2004

[6] : Kaplansky, *Commutative Rings*, Revised Edition, University of Chicago Press, Chicago, 1974.

[7] :T.G. Lucas, *The diameter of a zero-divisor graph*, J. Algebra 301 (2006).

[8] : S.P. Redmond, *Central sets and radii of the zero-divisor graphs of commutative rings*, Comm. Algebra 34 (2006), 2389–2401.

[9] :S.P. Redmond, *On zero-divisor graphs of small finite commutative rings*, Discrete Math. 307 (2007), 1155–1166.

[10] :S.P. Redmond, *The zero-divisor graph of a non-commutative ring*, Internat. J. Comm. Rings 1 (4) (2002), 203–211.

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